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Abstract: Every business seeks to correctly anticipate how much of which products it must manufacture, and where to deliver them, to satisfy the requirements of its customers, as well as its own requirements for growth and profitability. Since the forecast of demand is the cornerstone of all other, subsequent planning, errors become very costly very quickly. This paper presents an optimization model for determining the smoothing constants and initial estimates of level, trend, and seasonality indices in Winter's exponential smoothing forecasting model. The objective is to minimize the sum of squared forecast errors. An Excel template is available for download from the author's website. The template has built-in macros to perform all calculations. Instructions on how to use the template are included in the template.

I. Introduction

The forecast of demand is the gateway linking operations with the marketplace. Through that gateway, supply-chain managers hope to see clearly enough to plan (hire, buy, borrow, etc.), the better to reduce wastes, control costs, avoid conflicts, increase revenues and profits. Prior planning avoids fire-fighting, prepares for crises, and decreases the need for slack resources (capacity, inventory, staffing) to deal with unpredictable demand. In short, effective forecasting helps stabilize operations.

Every business seeks to correctly anticipate how much of which products it must manufacture, and where to deliver them, to satisfy the requirements of its customers, as well as its own requirements for growth and profitability. Since the forecast of demand is the cornerstone of all other, subsequent planning, errors become very costly very quickly. Accordingly, companies try hard to influencing the demand for their products, in the hope of exerting at least some control.

Forecasting models differ from one another according to whether they are qualitative or quantitative, simple or sophisticated. Qualitative forecasting methods rely on the subjective judgment and opinions of one or more individuals, about the likely course of future trends, tastes, technical changes, etc. These methods are useful in forecasting demand when there is little or no data to support quantitative methods (e.g., new products, new markets), and where it is believed that past relationships aren't good indication of the future.

Quantitative forecasting methods, on the other hand, involve some kind of mathematical modeling. They have stringent data requirements – more data points, or numerical data measured in a way that allows meaningful calculations (e.g., using interval or ratio scales.) Time series analysis is a common forecasting technique in supply chain management. As their name suggest, time series analysis deals with time-based series of values of the attribute of interest, such as quantity sold. The method postulates that: (1) the observed series is the combined result of multiple phenomena; (2) the series therefore can be "decomposed" into its constituent components, i.e., level, trend, seasonality and cycle; and (3) the pattern in the past will continue into the future, thereby justifying extrapolation beyond the historical data.

Time series models include, among many others, the method of moving averages, exponential smoothing (e.g., the Winter's Model), Fourier series analysis, Box-Jenkins auto-regressive moving averages (ARIMA), and time series decomposition.

This paper presents an optimization model for determining the smoothing constants and initial estimates of level, trend, and seasonality indices in exponential smoothing forecasting. The objective is to minimize the sum of squared forecast errors. An Excel template is available for download at www.csupomona.edu/~hco/ManagementScience/00-Winters.xls. The template has built-in macros to perform all calculations. Instructions on how to use the template are included in the template.

II. The Winter'S Model

The Winter's model is a seasonally-adjusted, trend-enhanced, exponential-smoothing forecasting model. Seasonally-adjusted trended forecasting uses historical data to derive a "base value L," a "trend value T," and a set of seasonal indices R1, R2 …Rm. Here m = number of periods that make up a complete seasonal pattern. For example, if we are dealing with monthly forecasts, then m = 12 months/year.

The demand forecast for period t+p is $F_{t+p} = R_{t-m+p}\,[L_t + (p)T_t]$, for p = 1, 2, … P. P is the length of the forecasting horizon. R_{t-m+p} is the corresponding seasonal index estimated from the preceding year, L_t is the level of demand estimated at period t, and T_t is the trend component of demand estimated at period t.

For an average period, the seasonal index = 1. The index is greater than 1 if the demand for the period is above

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average. On the other hand, an index less than 1 means that the demand for the period is lower than average.

To make seasonally-adjusted trended forecasts, we first have to find a way to derive the "base value L," the "trend value T," and the set of seasonal indices R. Then using the latest "base value," "trend value," and "seasonal indices" to generate our forecasts. For example, the forecast for month 1 (t = 1) is

$F_1 = R_{-11}\,(L_0+T_0)$

Note that R_{-11} is the seasonal index estimate at t = m-1 = 12-1 =-11. L_0 and T_0 are the level and trend components estimates at t = 0 (the preceding month).

The Winter's model consists of three revision equations, each equation for each of the three components of the time series:

Base value: $\alpha\left(A_1 / R_{-11}\right)+(1-\alpha)(L_0+T_0)$

Seasonal index: $\gamma\left(A_1 / L_1\right)+(1-\gamma)(R_{-11})$, and

Trend component: $\beta(SA_1 - SA_0)+(1-\beta)(T_0)$

$0 \le \alpha, \beta, \gamma \ge 1$ are the smoothing constants for the level, trend, and seasonality components of the time series.

In the following section, we will demonstrate the method of seasonally-adjusted trended forecasting. The process consists of the following steps:

1.Enter historical demand data.

2.Assume values of the base, the trend, the seasonal indices, and the smoothing constants.

3.Enter the revision and forecasting equation for the initial period. Copy and paste all the way to the row where the last demand data is.

4.Compute some measure of forecast errors.

5.Use Excel's Solver tool to minimize the forecast error, by changing our initial assumptions about the base value, the trend value, the seasonal indices, and the smoothing constants.

Use the latest "base value," "trend value," and "seasonal indices" to generate our forecasts.

III. Optimization Model

Supposed we have collected 60 months of data on demand for air conditioners. We want to forecast the demand for the next 12 months:

	A	B	C
1			
2		Data Range =	60
3			
4			Smoothing
5			Constants
6		α =	0.200
7		β =	0.200
8		γ =	0.200
9			
10		Demand	
11		1,037	
12		1,070	
13		1,116	
14		1,173	
15		1,337	
16		1,369	
17		1,285	
18		1,438	
19		1,382	
20		1,248	
21		1,254	
22		1,170	
23		1,216	
24		1,390	
25		1,498	
26		1,469	
27		1,600	
28		1,800	
29		1,812	
30		1,769	
31		1,636	
32		1,591	

1. The first step is to decide the values of the smoothing constants α, β, and γ. Additionally, we need the initial values of the level, trend and seasonal components. Let us arbitrarily assume α = β = γ = 0.2. We make reasonable assumptions about the initial values of the level, trend and seasonal components. This is shown below:

	A	B	C	D	E
1					
2		Data Range =	60		
3					
4			Smoothing		
5			Constants		
6		α =	0.200		
7		β =	0.200		
8		γ =	0.200		
9					
10		Demand	Level	Trend	Season Idx
11					1.000
12					1.000
13					1.000
14					1.000
15					1.000
16					1.000
17					1.000
18					1.000
19					1.000
20					1.000
21					1.000
22			985	52	1.000
23		1,037			
24		1,070			
25		1,116			
26		1,173			
27		1,337			
28		1,369			
29		1,285			
30		1,438			
31		1,382			
32		1,248			

Notice that we have inserted 12 rows after row 10. We need 12 rows for the 12 seasonal indices (Column E). In the worksheet above, we assumed that all seasonal indexes are equal to 1. The level component (cell C22) is 985 and the trend component (cell D22) is 52. Don't worry about making bad assumptions. Later, we will use an Excel tool called **Solver** to change them.

2. Write the Winter's Equations for the first period (i.e., for t =1). These are:

$$\alpha\left(A_t/R_{t\text{-}m}\right)+(1-\alpha)(L_{t-1}+T_{t-1})$$

$$\beta\left(L_t-L_{t\text{-}1}\right)+\left(1-\beta\right)\left(T_{t-1}\right)$$

$$\gamma\left(A_t/L_t\right)+(1-\gamma)(R_{t-m})$$

The corresponding Excel formulae are:
C23=$C$6*(B23/E11)+(1-$C$6)*(C22+D22)
D23=$C$7*(C23-C22)+(1-$C$7)*D22, and
E23=$C$8*(B23/C23)+(1-$C$8)*E11

The forecast for the first period is F23=(C22+D22)*E11, ,i.e., the sum of the level and trend component, multiplied by the first seasonal index.

The forecast error for the first period is G23=B23-F23

3. Copy all formulae and paste all the way down to the last row of demand (row 82).:

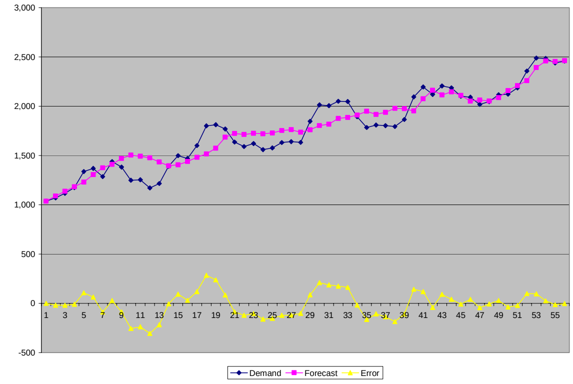
	A	B	C	D	E	F	G
1							
2		Data Range =	60				
3							
4			Smoothing				
5			Constants				
6		α =	0.200				
7		β =	0.200				
8		γ =	0.200				
9							
10		Demand	Level	Trend	Season Idx	Forecast	Error
11					1.000		
12					1.000		
13					1.000		
14					1.000		
15					1.000		
16					1.000		
17					1.000		
18					1.000		
19					1.000		
20					1.000		
21					1.000		
22			985	52	1.000		
23		1,037	1,037	52	1.000	1,037	0
24		1,070	1,085	51	0.997	1,089	-19
25		1,116	1,132	50	0.997	1,136	-20
26		1,173	1,181	50	0.999	1,182	-9
27		1,337	1,252	54	1.014	1,230	107
28		1,369	1,319	57	1.008	1,306	63
29		1,285	1,357	53	0.989	1,375	-90
30		1,438	1,416	54	1.003	1,410	28
31		1,382	1,452	51	0.990	1,470	-88
32		1,248	1,452	40	0.972	1,503	-255

4. Enter formulae for sum of squared error and the average (Mean) error. Also, enter the formula for I4=AVERAGE(F11:F22); I4 is the average of the first 12 month's seasonal indices.
L24==SUMSQ(G23:G82)
L25=AVERAGE(G23:G82), and
L26=AVERAGE(E11:E22).

L24 | fx =SUMSQ($G$23:INDEX(G:G,$C$2+22))

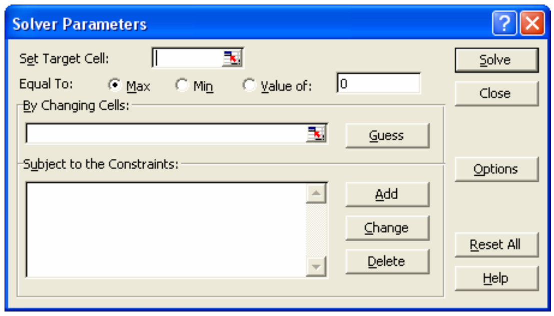
	H	I	J	K	L	M
22						
23		Summary Statistics				
24		Sum of Squared Errors			917,831	
25		Mean Error			-11	
26		Average of Season Idx			1.000	

The sum of squared error is 917,831. The mean squared error (MSE) = 917,831/59 =15556. Figure 1 below shows a graph of the actual and forecasted demand:



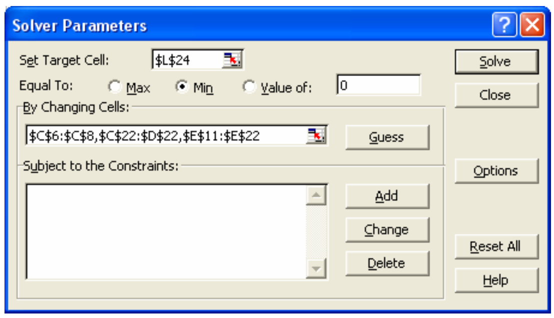
5. In step 1, we assumed, quite arbitrarily, the values of the smoothing constants, and the initial values of the level, the trend, and seasonal components of the time series. In this step, we will use a tool in Excel called Solver, to minimize the sum of squared forecast errors, by changing these assumed values.

Click Tools on the top of the Excel worksheet. From the Tools menu, select Solver, and the following Solver Parameter appears:



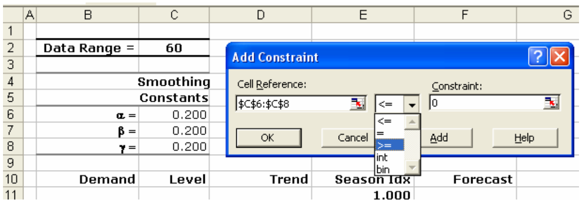
Starting from the top, enter L24 in the 'Set Target Cell' box. L24 is the sum of squared forecast errors. Next, click on the Min button to tell Solver that you want to minimize L24.

In the 'By Changing Box,' enter the cells where you find the values of the smoothing constants, and the initial values of the level, the trend, and seasonal components of the time series. These are: C6:C8 for the smoothing constants, C22:D22 for the level and trend components, and E11:E22 for the seasonal indices.



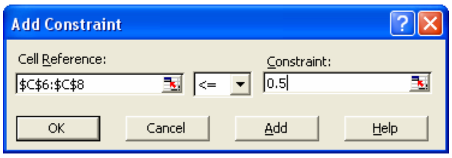
Note that Excel automatically add the ‘$’ sign to the cell references above.

Now, click Add to introduce some constraints. To make sure that the smoothing constants are non-negative, enter C6:C8 in the Cell Reference box, choose the greater-than-or-equal-to symbol ≥, and enter 0 in the constant box. This is shown below:



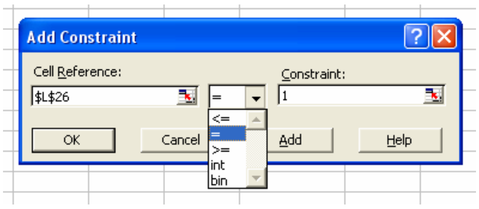
Note that instead of specifying 0 as the lower-bound for the smoothing constant, you may want to specify a small constant, say 0.01.

Similarly, we may want to specify an upper-limit for the values of the smoothing constant. Click Add to introduce another constraint:

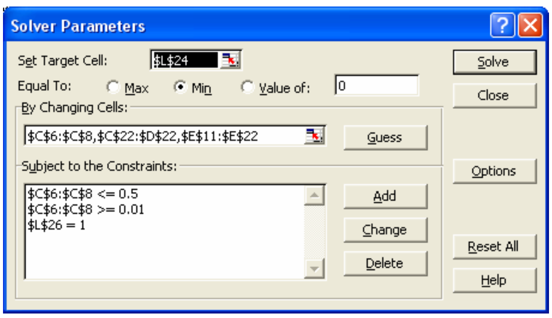


Here, we specify that the smoothing constants must not exceed 0.5.

Finally, add a constraint to make sure that the average of the seasonal indices =1. This is shown below:

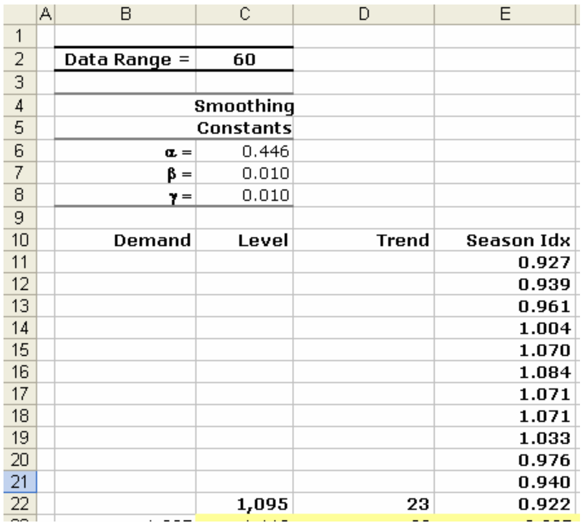


The completed Solver Parameter menu is shown below:

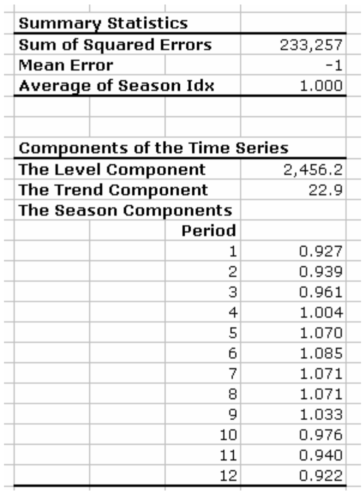


Here the lower-and-upper bound for the smoothing constants have been set at 0.01 and 0.5, respectively. Click ‘Solve’ to minimize the sum of squared forecast errors.

6. Optimized Worksheet

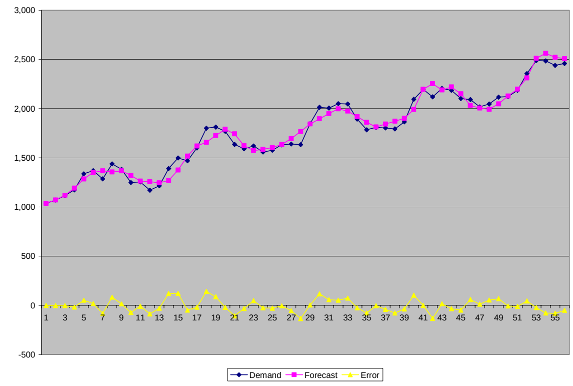


	A	B	C	D	E
1					
2		Data Range =	60		
3					
4			Smoothing		
5			Constants		
6		α =	0.446		
7		β =	0.010		
8		γ =	0.010		
9					
10		Demand	Level	Trend	Season Idx
11					0.927
12					0.939
13					0.961
14					1.004
15					1.070
16					1.084
17					1.071
18					1.071
19					1.033
20					0.976
21					0.940
22			1,095	23	0.922



Summary Statistics			
Sum of Squared Errors			233,257
Mean Error			-1
Average of Season Idx			1.000
Components of the Time Series			
The Level Component			2,456.2
The Trend Component			22.9
The Season Components			
		Period	
		1	0.927
		2	0.939
		3	0.961
		4	1.004
		5	1.070
		6	1.085
		7	1.071
		8	1.071
		9	1.033
		10	0.976
		11	0.940
		12	0.922

The sum of squared error is 233,257. The MSE = 233,257/59 =3954. Using Solver, we were able to reduce the MSE by 75%. Figure 2 below shows a graph of the actual and forecasted demand:



The components of the time series shown here are the latest “base value,” “trend value,” and “seasonal indices” to generate our forecasts. These values can be found at the bottom of the spreadsheet.

The forecasts are:

Jan	[2,456.4 + (1) * (22.9)] * 0.927 =	995
Feb	[2,456.4 + (2) * (22.9)] * 0.939 =	1.023
Mar	[2,456.4 + (3) * (22.9)] * 0.965 =	1.065
Apr	[2,456.4 + (4) * (22.9)] * 1.004 =	1.145
May	[2,456.4 + (5) * (22.9)] * 1.070 =	1.303
Jun	[2,456.4 + (6) * (22.9)] * 1.085 =	1.215
Jul	[2,456.4 + (7) * (22.9)] * 1.071 =	1.239
Aug	[2,456.4 + (8) * (22.9)] * 1.071 =	1.225
Sep	[2,456.4 + (9) * (22.9)] * 1.033 =	1.089
Oct	[2,456.4 + (10) * (22.9)] * 0.976 =	1.006
Nov	[2,456.4 + (11) * (22.9)] *0.940 =	967
Dec	[2,456.4 + (12) * (22.9)] *0.922 =	928

IV. Final Note

Using Solver, we were able to reduce the MSE by 75%. In the illustration above, we allowed Solver to change the smoothing constants, and the initial values of the level, the trend, and seasonal components. Sometimes, we are better off using our judgment to decide on the appropriate values of the smoothing constants.

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